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INVESTIGATION OF THE GENERAL CIRCULATION OF THE ATMOSPHERE - 1 JULY-30 SEPTEMBER 1948 (2)

VICTOR P. STARR SEPT '48 10PP

USAF CONTR. NO. W28-099 AC-406

METEOROLOGY (30) ATMOSPHERE - CIRCULATION
ATMOSPHERIC STRUCTURE ATMOSPHERE - PHYSICS
AND PHYSICS (2)

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Report No. 2

INVESTIGATION OF THE GENERAL CIRCULATION OF THE ATMOSPHERE

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30 September 1948

Contract W28-099 ac-406
1 July 1948 - 30 September 1948

Personnel Changes

Dr. Abdul J. Abdullah is now devoting twenty per cent. (20%) of full time to this project.

Business

A Marchant Calculating Machine, Model ACR8M, has been purchased.

Abstract

The variations of absolute angular momentum about the earth's axis during January 1946 have been studied. The data were obtained from northern hemisphere synoptic maps at sea level, 700 millibars, and 500 millibars. It was found that from 15°N to 35°N, addition of angular momentum to the atmosphere by friction accompanied depletion of angular momentum by horizontal transport of relative angular momentum, i.e., angular momentum due to motion relative to the earth's surface. From 35°N to 65°N, accumulation of angular momentum by horizontal transport accompanied removal of angular momentum from the atmosphere by friction. These results are in agreement with theory.

An auxiliary study has indicated that the true northward transport of relative angular momentum is usually larger than the northward transport as computed under the assumption that the wind is geostrophic. Another auxiliary study has shown that the mountain effect, i.e., the torque exerted by a mountain range upon the atmosphere when the atmospheric pressures on the west and east sides of the range are different at the same altitude, is as important as friction in exchanging angular momentum between the atmosphere and the earth's surface.

In a previous report it was proposed that values be obtained for the northward transport of absolute angular momentum about the earth's axis. These values were to be compared with values of the exchange of momentum between the atmosphere and the earth's surface. Such values have been computed, primarily by Widger, for the month of January 1946.

Procedure: The data are obtained from daily synoptic maps of the northern hemisphere. The sea level and 500 millibar maps are from the Northern Hemisphere Historical Weather Maps of the Air Weather Service. The 700 millibar maps are photostat copies of maps analyzed by the Air Weather Service.

The sea level pressure is tabulated at each intersection of a five-degree meridian with a five-degree parallel, for each day, within the limits of the analysis. The heights of the 700 millibar and 500 millibar levels are tabulated at the same network of points.

Since observed winds do not appear on the maps at a regular network of points, the geostrophic wind is used instead of the actual wind. An auxiliary investigation has been made to determine the accuracy of this approximation.

Let R = radius of earth

ω = angular speed of earth's rotation

g = acceleration of gravity

λ = longitude

ϕ = latitude

p = pressure

z = height (of a constant pressure surface)

ρ = density

u = eastward component of wind

v = northward component of wind

u_g = eastward component of geostrophic wind

v_g = northward component of geostrophic wind

$$\text{Let } \Delta_x p(\lambda, \phi) = p(\lambda + 5^\circ, \phi) - p(\lambda - 5^\circ, \phi)$$

$$\Delta_y p(\lambda, \phi) = p(\lambda, \phi + 5^\circ) - p(\lambda, \phi - 5^\circ)$$

and let similar relations define $\Delta_x z$ and $\Delta_y z$

Then approximately,

$$u_g = - \frac{9 \Delta_y p}{\pi R \omega \rho \sin \phi}, \quad v_g = \frac{9 \Delta_x p}{\pi R \omega \rho \sin \phi \cos \phi}$$

on a constant altitude surface, and

$$u_g = - \frac{9 g \Delta_y z}{\pi R \omega \sin \phi}, \quad v_g = \frac{9 g \Delta_x z}{\pi R \omega \sin \phi \cos \phi}$$

on a constant pressure surface.

The absolute angular momentum per unit volume, given by

$$\rho(u + \omega R \cos \phi) R \cos \phi$$

consists of the angular momentum $\rho \omega R^2 \cos^2 \phi$ due to the earth rotation, hereafter called ω-momentum, and the angular momentum $\rho u R \cos \phi$ due to motion relative to the earth's surface, hereafter called relative momentum. Since the transport of ω-momentum across a given latitude is proportional to the mass transport across that latitude, there can be no net long term transport of ω-momentum. The net long term absolute momentum transport therefore consists of a transport of relative momentum.

The transport of relative momentum per unit time per unit altitude across a given latitude ϕ is

$$\int_0^{2\pi} \rho u v R^2 \cos^2 \phi \, d\lambda$$

In computations the above integral is replaced by a summation of 72 terms, one for every five degrees of longitude, and the wind is replaced by the geostrophic wind. The density is assumed to be the normal density for the given month, latitude, and altitude. The transport is thus

$$-\frac{9\cos\phi}{4\pi\omega^2\rho\sin^2\phi}\sum\Delta_x p\,\Delta_y p$$

at sea level, and

$$-\frac{9g^2\rho\cos\phi}{4\pi\omega^2\sin^2\phi}\sum\Delta_x z\,\Delta_y z$$

at the constant pressure surfaces.

The monthly total transport across a latitude circle is estimated as follows: the transports per unit altitude computed from the sea level, 700 millibar, and 500 millibar maps are assumed to be the transports per unit altitude throughout the layers from 0 to 1.5 kilometers, 1.5 to 4.5 kilometers, and 4.5 to 7.5 kilometers respectively. The transport above 7.5 kilometers cannot be computed because no data are available except in restricted regions. The transport per unit time computed from a given map is assumed to be the transport per unit time throughout the day of the map.

The interaction between the atmosphere and the earth's surface consists of a friction effect and a mountain effect. The mountain effect is the torque exerted by a mountain range upon the atmosphere when the atmospheric pressures on the west and east sides of the range are different at the same altitude. An auxiliary investigation is being made to determine the importance of the mountain effect.

The eastward component of the frictional force per unit surface area is assumed to be given by

$$\kappa \rho u \sqrt{u^2 + v^2}$$

where κ is a dimensionless constant, assumed to be 3×10^{-3}. The torque per unit northward distance around an entire latitude circle is therefore

$$\int_0^{2\pi} \kappa \rho u \sqrt{u^2 + v^2} \; R^3 \cos^2 \phi \, d\lambda$$

The surface wind vector is assumed to be 0.6 times the surface geostrophic wind vector. The torque per unit northward distance is thus

$$- \frac{81 \kappa R \cos^2 \phi}{100 \pi \omega^2 \rho \sin^2 \phi} \Delta_y p \sqrt{(\Delta_y p)^2 + \frac{1}{\cos^2 \phi} (\Delta_x p)^2}$$

<u>First Auxiliary Study</u>: An investigation of the accuracy of the geostrophic wind approximation for the computation of the transport of momentum was made primarily by Lorenz. The study is based upon data collected by the Pressure Change Project at MIT (sponsored by the Office of Naval Research, Contract N5ori-78, Task IV). The data consist of simultaneous values of the observed wind (by rawin or rabal) and the geostrophic wind at stations in the United States. Unfortunately, such data for the remainder of the northern hemisphere are not available.

The data used contain 267 observations during January 1946 and 283 observations during December 1946 - January 1947. For each observation, the products uv and $u_g v_g$ are computed. The sums Σ uv and $\Sigma u_g v_g$ are obtained for each period.

For January 1946,

$$\Sigma u_g v_g < \Sigma u v < 0,$$

and for December 1946 - January 1947,

$$0 < \sum u_g v_g < \sum u\, v$$

During each period, $u\,v$ is on the average somewhat smaller than $u_g v_g$ when $u_g v_g > 0$, and $u\,v$ is on the average considerably greater algebraically than $u_g v_g$ when $u_g v_g < 0$. Investigation of the observations at 500 millibars suggests similar results.

If these results, which include portions of the hemisphere where $\sum u_g v_g > 0$ and portions where $\sum u_g v_g < 0$, are typical of the entire hemisphere, they indicate that for an entire latitude circle at middle latitudes,

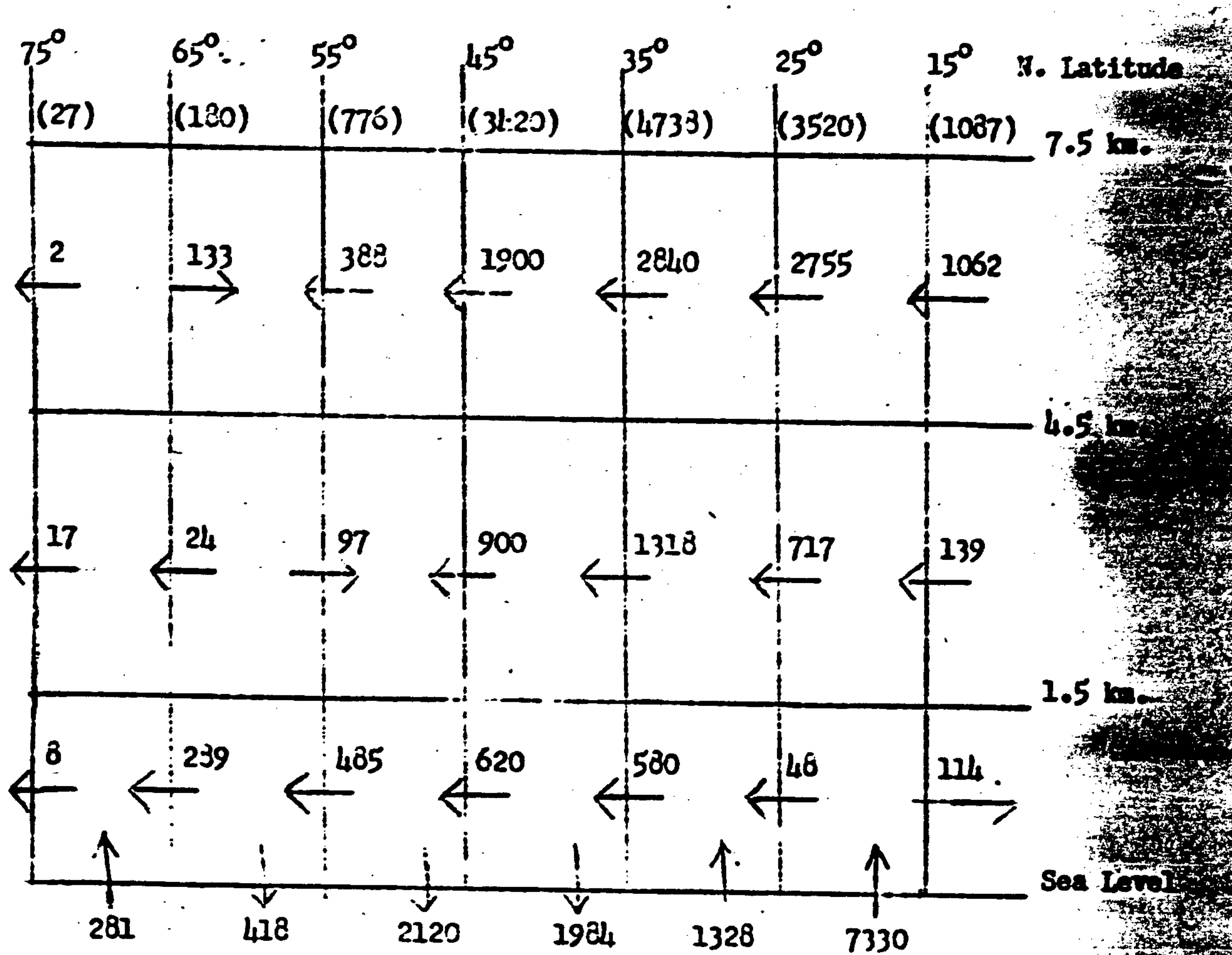
$$\int_0^{2\pi} u\, v\, d\lambda > \int_0^{2\pi} u_g v_g\, d\lambda$$

The momentum transport $\int_0^{2\pi} \rho\, u\, v\, R^2 \cos^2 \phi\, d\lambda$ depends largely upon $\int_0^{2\pi} u\, v\, d\lambda$.

In general, the monthly values of momentum transport, computed from the northern hemisphere maps with the geostrophic wind approximation, are positive. It was arbitrarily decided to multiply these computed values by the factor 1.5, in the hope of obtaining more accurate values of the momentum transport.

Results: The results of the investigation are summarized in Figure 1.

FIGURE 1



75°
65°
55°
45°
35°
25°
15°
N. Latitude
(27)
(180)
(776)
(3120)
(4738)
(3520)
(1087)
7.5 km.
2
133
388
1900
2840
2755
1062
4.5 km.
17
24
97
900
1318
717
139
1.5 km.
8
239
485
620
580
48
114
Sea Level
281
418
2120
1984
1328
7330

In Figure 1, the horizontal arrows with the accompanying numbers show the horizontal transport of relative angular momentum across the corresponding latitude, throughout the corresponding height interval. The numbers in parentheses give the total transport of relative momentum across the corresponding latitude. The vertical arrows with the accompanying numbers show the total frictional transfer of momentum from the earth's surface to the atmosphere between the corresponding latitudes. All the numbers, when multiplied by 10^{29}, become values in c.g.s. units.

In general, the horizontal transport of relative momentum is directed northward. South of 35°N, addition of momentum to the atmosphere by friction accompanies depletion of momentum by horizontal transport, while from 35°N to 65°N, accumulation of momentum by horizontal transport accompanies removal of momentum from the atmosphere by friction. These observations are in agreement with the theoretical conclusions of Starr (see Journal of Meteorology, April 1948).

It is not to be expected that the depletion or accumulation of relative momentum by horizontal transport will exactly balance the addition or removal of momentum by friction, for several reasons. The computations described above involve many approximations. It has not been possible to determine the transport of relative momentum above 7.5 kilometers. The net horizontal transport of ω-momentum, and the net local change of absolute momentum, while unimportant for very long periods, may be important for periods as short as one month. Finally, the mountain effect has been neglected.

Second Auxiliary Study: An investigation of the importance of the mountain effect has been made primarily by White. This investigation has thus far determined only the normal mountain effect for January.

Normal January pressures for the northern hemisphere are obtained from figures of Shaw at sea level, and from USAAF revised figures (1944) at 10,000 feet. Simplified topographic profiles for each five-degree belt of latitude, based on the Physiche Weltkarte of Hermann Haack, are constructed for the northern hemisphere. Each mountain range is broken into layers one kilometer thick. Pressure differences Δp across each layer of each mountain range are obtained from the normal January pressures. The pressures in each layer are interpolated or extrapolated from the sea level and 10,000 foot pressures. The torque per unit latitude exerted by the mountains on the atmosphere is

$$1 \text{ Kilometer} \cdot R \cos \phi \sum \Delta p \cdot,$$

the summation extending over all mountain ranges and all layers.

It is found that the normal January mountain effect varies with latitude in much the same way as the friction effect. South of 40°N, and north of 60°N, within the limits of the data, the mountain effect adds momentum to the atmosphere. Between 40°N and 60°N it removes momentum from the atmosphere. The mountain effect is strongest in the belt of westerlies between 45°N and 50°N, and in the belt of easterlies between 25°N and 35°N. The mountain effect and the friction effect are of the same order of magnitude.

It must be concluded that the mountain effect not only is not negligible, but is sometimes more important than the friction effect both in removing momentum from the atmosphere and in adding momentum to it.

Additional Studies: A number of purely theoretical features of the general circulation have been investigated by Starr, Abdullah, and Lorenz. Some preliminary results are expected in the near future.

Future Investigations: Investigation of the variations of absolute angular momentum during January 1946 is to be continued. The mountain effect during this month is to be computed. The local variations of absolute angular momentum, and the northward transport of ω-momentum, are also to be determined. The variations of absolute angular momentum during January 1946 are also to be studied on a day-to-day basis. The entire general program is to be extended to other months.